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MATHEMATICAL FORMAL MODELS FOR THE LEARNING OF PHYSICS: THE ROLE OF AN HISTORICAL EXAMPLE

Giorgio T. Bagni,
Department of Mathematics,
University of Roma "La Sapienza", Italy

Abstract History of Sciences is an important tool for Didactics: in this paper we propose the introduction of the concepts of work and of kinetic energy by an example based upon G. M. Ciassi's work (1677). Frequently historical development of a concept is not suitable in order to plan curricula; however sometimes there is an analogy between the stages of the historical development and corresponding educational stages. Of course, processes of teaching-learning take place nowadays: so educational work can be based upon the results achieved in the full historical development and the History makes it possible to point out mathematical formal models that can be used in Didactics of Physics by analogy. This correspondence is an important tool for teachers: of course a deep epistemological skill is needed and this is a matter related to teacher training.

The History of Sciences is an important tool for Didactics: in fact in this paper we study the introduction of the concepts of work and of kinetic energy according to an historical example. Of course, first of all it is worth noting that frequently historical development of a concept is not suitable in order to plan curricula, although sometimes we can point out analogies between stages of the historical development and corresponding educational stages: educational work can be based upon the results achieved in the full historical development, and we particularly underline that the History of Sciences makes it possible to point out mathematical formal models that can be used in Didactics of Physics by analogy.

The 17th century is characterised by a great cultural vivacity. The question about which the debate regarding the "vis viva" took place was the following: what is the physical magnitude that causes the motion? According to René Descartes (1596-1650) (Michieli, 1949), such magnitude would be the "quantitas motus", i.e. the product of the mass of the considered body and its speed. On the contrary, Gottfried Wilhelm Leibniz (1646-1716) published the paper entitled *Brevis demonstratio erroris memorabilis Cartesii, et aliorum circa legem naturalem, secundum quam volunt a Deo eamdem semper quantitatem motus conservari; qua et in re mechanica abutuntur* (1). He described a simple experiment and concluded that "quantitas motus" cannot be considered the cause of the motion: so it is necessary to define a new "vis motrix".

Such "vis motrix" was introduced by Leibniz himself in the work *Specimen dynamicum pro admirandis naturae legibus circa corporum vires et mutuas actiones detergendis et ad suas causas revocandis* (1695); but Leibnitian "vis motrix", or "vis viva", was not clearly defined: it is not proportional to the speed of the considered body (as "quantitas motus" is), but it is proportional to the square of such speed (2).

Of course, Leibnitian ideas too cannot be considered totally correct, from a modern point of view: Leibniz considered implicitly his "vis motrix" as a real force: the modern concept of work was still ignored.

In the development of the question about the "vis motrix" we must consider Gian Maria Ciassi (1654-1679), who wrote *Tractatus physicomathematicus*, published in Venice in 1677 (Ciassi, 1677; Nicolai, 1754; Pellizzari, 1830; Rambaldi, 1863; Michieli, 1949; Bagni, 1991, 1992 and 1993); in this work we can find some interesting notes.

Ciassi was born in Treviso, Italy, on march 20, 1654, and studied in Padua (Favaro, 1917); he died, only 25 years old, in Venice. His physical work was based upon the use of mathematical tools, e.g. geometric proofs. In *Tractatus physicomathematicus* Ciassi's aim is the justification of some statements exposed in *Meditationes de natura plantarum*, Ciassi's previous work. The Author compares the situation of a lever to the study of the equilibrium of a fluid in communicating vessels, and surely this is the most interesting part of Ciassi's *Tractatus*.

Let us report some remarks, in Latin original text: "Immo haec ipsa altitudinis linearum a motis corporibus descriptarum reciprocatio cum gravitate ipsorum prior causa est, aequalis momenti, quod Galileus non advertit. Etenim corpus cum alio in hac reciprocatione constitutum unam tantum unciam gravitans, ut elevetur ad quatuor pollices, eandem vim requirit, ac corpus gravitans quatuor uncias, ut elevetur ad unum pollicem tantum. Puta ut corpus G unam tantum unciam gravitans attollatur per lineam EA, cuius altitudo sit quattuor pollicum; requiritur eadem vis, ac ut corpus F quatuor uncias gravitans attollatur per lineam DB, cuius altitudo sit tantum unius pollicis. Quia scilicet cum in altitudine lineae EA sint quatuor partes, quarum unaquaeque est aequalis altitudini DB totius; licet ad elevandam corpus G ad singulas harum quatuor partium requireretur alias tantum quarta virium pars, quae requiritur in elevatione corporis F ad equalem altitudinem totius DB; in omnibus tamen simul quatuor partibus EA requiritur quadrupla vis; quia quater ea quarta virium pars replicatur" (Ciassi, 1677, pp. 57-59).

So Ciassi states that a body G, in the point E, to be lifted up to the point A requests "vis" as a body F in D lifted up to the point B if and only if the weights G and F are inversely proportional to GC and FC, so to the virtually covered segments AE and BD.

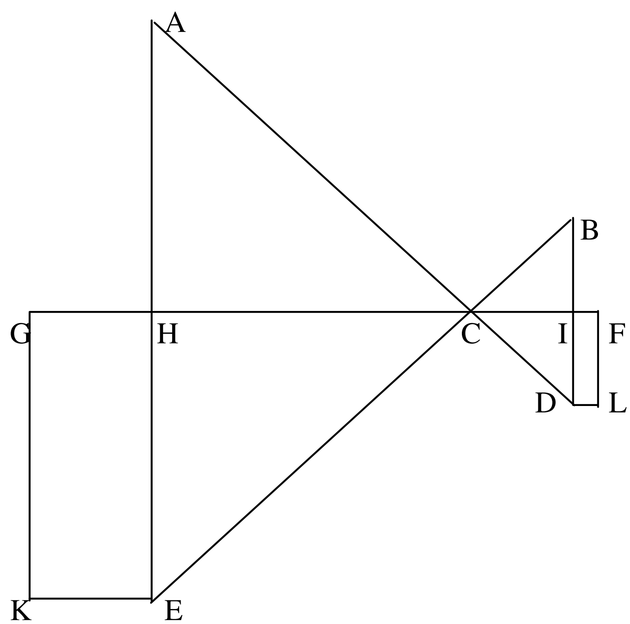

Figure 1 (Ciassi, 1677, p. 54)
The Author geometrically proves that: AC : CD = AE : BD
so, being: GK = HE = AE/2 and FL = ID = BD/2,it follows: AC : CD = GK : FL

In order to appreciate the real importance of Ciassi's conclusions, let us underline that the statement that P and P' are inversely proportional to h and h' is equivalent to the statement of a similar proportionality to the squares of the speeds v, v' referred to respective motions of considered points (in fact h is proportional to the square of the speed, being $2gh$). So Ciassi's statement implies the proportionality of the "vis" (of course, today, we should refer to kinetic energy) acquired by a dropping body to the square of its speed and it can be considered in the theoretic frame of the problem of the "vis viva", according to Leibnitian point of view.

It is important to underline that the publication of Ciassi's *Tractatus physicomathematicus* took place in 1677, so nine years before the publication in "Acta Eruditorum Lipsiae" (1686) of the celebrated Leibnitian work about the "vis motrix": it could be interesting to investigate if Leibniz knew, in 1686, Ciassi's research and results. Let us compare, for instance, some words by the considered Authors:

Ciassi *in 1677* wrote:

"A body weighing one ounce, considered with another body in such lever, and lifted up to four inches requests the same work requested by a body weighing four ounces lifted up to one inch" ([2]).

Leibniz *in 1686* wrote:

"I suppose that the same work is necessary either in order to lift up a body weighing one pound to the height of four yards, or in order to lift up a body weighing four pounds to the height of one yard" ([3]).

Of course, in this case, the similarity can be referred just to a secondary statement; moreover, previously mentioned Ciassi's main result, too, is not accompanied with clear references to the opposition between Descartes' ideas and Leibnitian solution of the problem of the "vis motrix".

So we don't want to give Gian Maria Ciassi full credit for the direct solution of the considered problem. However, Ciassi's studies can be considered surely important and historically interesting.

According to Y. Chevallard's terminology, as we noticed previously, we can state that the History of Sciences is an important tool for the *transposition didactique*. The well known *triangle of Chevallard* visualises a really frequent situation: the academic knowledge (the so-called *savoir savant*) is sometimes far from the process of teaching-learning (Chevallard, 1985). So we must «draw up» the *savoir savant* to the classroom practice, to the process of teaching-learning by the *transposition didactique*: it can be achieved by the use of some historical examples, too. Once again we must remember that the historical development is not always suitable in order to plan curricula, although sometimes it is possible to point out an analogy between the stages of the historical development and educational stages.

As regards the *savoir savant*, the historical development of a concept can be considered as the sequence of (at least) two stages: an early, intuitive stage and a mature stage; in the early stage the focus is mainly operational; the structural point of view is not a primary one. From the educational point of view, a similar situation can be pointed out (Sfard, 1991): in the early stage pupils approach concepts by intuition, without a full comprehension of the matter; then the learning becomes better and betters, until it is mature. Of course, processes of teaching-learning take place nowadays, after the full development of the *savoir savant*. So the *transposition didactique*, whose goal is initially a correct development of intuitive aspects, can be based upon the results achieved in the mature stage, too, of the development of the *savoir savant*.

Moreover the process of teaching-learning and the *transposition didactique* must consider that pupils' reactions are sometimes similar to corresponding reactions noticed in the History; this correspondence and, of course, the knowledge of historical examples themselves are important tools for teachers: epistemological skill is needed, and this is a matter related to teacher training.

From the educational point of view, it is worth noting that the quoted historical example deals with the analogy between different situations, as geometric features of a lever and energy or work. Of course, as regards analogical reasoning, we must underline that the really different propensity for self-correction should be considered, e.g. when we compare research scientists and young students: frequently scientists employ analogical reasoning in formulation of a conjecture, whose soundness must be verified; on the other hand, generally students do not perform this meta-discursive monitoring (for instance, some mathematical examples are discussed in: Bagni, 2000) ([4]).

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Notes

(1) Let us quote Leibniz himself: "... suppono, primo corpus cadens ex certa altitudine acquirere vim eousque rursus assurgendi, si directio eius ita ferat, nec quicquam externorum impediat... Suppono item secundo, tanta vi opus esse ad elevandum corpus A unius librae usque ad altitudinem CD quatuor ulnarum, quanta opus est ad elevandum corpus B quatuor librarum, usque ad altitudinem EF unius ulnae... Hinc sequitur corpus A delapsum ex altitudine CD praecise tantum acquisivisse virium, quantum corpus B lapsum ex altitudine EF. Nam corpus A postquam lapsu ex C pervenit ad D, ibi habet vim reassurgendi usque ad C, per suppos. 1, hoc est vim elevandi corpus unius librae (corpus scilicet proprium) ad altitudinem quatuor ulnarum. Et similiter corpus B postquam lapsu ex E pervenit ad F, ibi habet vim reassurgendi usque ad E, per suppos. 1, hoc est vim elevandi corpus quatuor librarum (corpus scilicet proprium) ad altitudinem unius ulnae. Ergo per suppos. 2, vis corporis A existentis in D, et vis corporis B existentis in E, sunt aequales". And Leibniz concludes: "itaque magnu m est discrimen inter vim motricem, et quantitatem motus, ita ut unum per alterum aestimari non possit" (Leibniz, 1768).

(2) Ciassi, 1677, p. 57. By "work" we translate Ciassi's "vis".

(3) Leibniz, 1768, III, pp. 180-181. By "work" we translate Leibnitian "vis".

(4) A clear educational problem consists in the uncertainty about the effects upon the learning of teachers' choices. This uncertainty concerns particularly the cognitive *transfer* (Feldman & Toulmin, 1976; D'Amore & Frabboni, 1996) that must be stim ulated by the teacher; and effects upon the learning must be carefully verified (D'Amore, 1999): so the use of historical examples can be useful to introduce some important topics: however their effectiveness must be carefully controlled in order to obtain a correct, full learning.

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