

# Functions: processes, properties, objects

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**Summary.** Some aspects of the learning of the function concept are investigated: as regard action views, object-oriented and property-oriented approaches, the role of representations is fundamental. Teacher's role in the institutionalization is important in order to make it possible the full reification.

## 1. Functions and their representations: from processes to objects

Several researches show that learning of the function concept is often facilitated by the early consideration of an action and its interpretation as a process (Briedenbach & Al., 1992): according to A. Sfard, the development of abstract mathematical objects is the product of the comprehension of processes (Sfard, 1989; although some researchers suggest models that are not strictly sequential: Slavit, 1997, p. 268; Artigue, 1998; the important notion of *procept* underlines symbols' roles: Gray & Tall, 1994).

In this theoretical framework, in order to consider an *action*, a *process* and, finally, an *object*, semiotic aspects are important; the distinction between an object and its representations is «a strategical point for the comprehension» (Duval, 1993, p. 37; Duval, 1995; suggestions to use representations introducing functions can be found from the late Eighties: Kaput, 1989). In order to obtain full learning it is not enough to have a development of (single) registers: their coordination is needed. Moreover, let us underline that there is not a single register of a given kind (for instance, a single visual register): in fact the nature of a register depends on the community of practice in question, so on various cultural frameworks; frequently a representation register is linked from other conceptual aspects (Tall & Al., 2001).

A. Sfard calls *reification* the passage from the consideration of a process to a conception properly referred to the mathematical object (however D. Slavit notices any «lack of clarity» when an *object-oriented* comprehension of a mathematical idea is stated: Slavit, 1997, p. 265, Thompson, 1994). The full and effective realization of the reification is important: if we force a structural point of view, we can cause the formation of dangerous pseudo-objects and misconceptions, in pupils' minds.

## 2. Functions and properties

As regards a *property-oriented* approach to function concept (Kieren, 1990), we must remember that it does not replace previous theories, but proposes a new interpretation of them (Slavit, 1997, p. 269). According to this approach, a function can be described with reference to its local and global properties; educational experience allows us to state that the study of properties is fundamental in order to characterize classes of functions. Clearly a *property-oriented* approach deals with pupils' ability to establish connections between representations (Monk & Nemirowsky, 1994), frequently with reference to technology (Ruthven, 1990). Different features of visual and symbolic representations can bring to different possibilities of such registers to be employed: this fact can constitute an obstacle, particularly if the coordination of representation registers is lacking. Experimental researches (Slavit, 1997, p. 272) pointed out that frequently pupils use either approaches based upon the consideration of a real correspondence (*action view*, *operational view*) or *property-oriented* approaches.

However these approaches, whose educational importance is primary, do not solve completely the problem of the reification: D. Slavit notices that there are no works proving whether a *property-oriented* approach effectively improves the development of an *object-oriented* conception of function (Slavit, 1997, p. 271).

## 3. Institutionalization and teacher's role

The passage from the early consideration of an action to the conception of a process (*interiorization*) is often referred to a particular situation: so pupils deal with a single example of the object that, in the future, will be generally considered as *function*. A *property-oriented* approach allows to point out features of a class of functions, in particular if the game is played with reference to representations. Then a difficulty in building the abstract object is the *generalization* (Eisenberg & Dreyfus, 1994).

Teachers play a primary role in the step from the consideration of a process to the building of an object: they, in didactic situations, verify that all elements that are going to constitute the *concept image*, and, later, the *concept definition*, keep their correct roles (Tall & Vinner, 1981, p. 152); then the teacher proposes to pupils the final generalization. The study of this step would be treated in further researches.

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