Chapter 8

Historical support for particular subjects

8.4   Detailed treatment of particular examples

8.4.1   Introducing complex numbers: an experiment

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The introduction of imaginary numbers is an important step in the high school mathematics curriculum (students 15-19 years old). High school students of 11 to 14 years old are often reminded about the impossibility of calculating the square root of negative numbers. However, at a later stage, they are asked to accept the presence of' the square root of -1, named i. This inconsistency can be a source of confusion.

On the other hand, we may consider the solution of cubic equations following the work of Niccolò Fontana (Tartaglia, 1500-1557), Girolamo Cardano (1501-1576), and Rafael Bombelli (1526-1573): imaginary numbers were not introduced via quadratic equations, but via cubic equations, an approach having a basic advantage. Their resolution does not take place entirely in the set of real numbers, but one of the final results is always real.

A recent study was motivated by this fact (**7.3.2**). In this research 97 high school students (aged 16-18), who did not know complex numbers, were interviewed (Bagni 1997). For the equation:

$$x^2+1 = 0$$

hence:

$$x = \pm i$$

only 2% accepted the solution, 92% rejected it and 6% did not answer. Afterwards, the solution of the cubic equation:

$$x^3 = 15x+4$$

namely:

$$x = (2+11i)^{1/3} + (2-11i)^{1/3}$$

so that:

$$x = (2+i) + (2-i) = 4$$

was accepted by 54%; 35% rejected it and 11% did not answer.

Under the same conditions, a similar test was then proposed to 52 students of the same age group, where the equations were presented in the reverse order: 41% accepted the solution of the cubic equation (25% rejected it and 34% did not answer). Immediately after that, the solution of the quadratic equation was accepted by 18% of the students, with only 66% rejecting it (16% did not answer). These experimental results suggest that teaching a subject by taking account of some basic facts in historical development may help students to acquire a better understanding of it (Weil 1978; Fauvel 1990; Swetz 1995).

References for § 8.4.1

Bagni, G.T. 1997. Ma un passaggio non è il risultato... L'introduzione dei numeri immaginari nella scuola superiore, *La matematica e la sua didattica* 2, 787-207

Fauvel, J. (ed.) 1990. History in the mathematics classroom, *The IREM papers*, Leicester, The Mathematical Association

Swetz, F.J. 1995. To know and to teach: mathematical pedagogy from a historical context, *Educational Studies in Mathematics* 29, 73-88.

Weil, A. 1978. History of mathematics: why and how, Letho, O. (ed.), *Proceedings of International Congress of Mathematicians*, I, Helsinki: Academie Scientiarum Fennica, 227-236.