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History of Mathematics and Didactics: Reflections on Teachers Education

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Abstract The role of history into mathematics teaching and learning and, in particular, into teachers education is discussed and some theoretical frameworks are sketched. We conclude that mathematical knowledge, didactical competence and historical knowledge cannot exist in isolation, and their connections reflect important epistemological assumptions. So such aspects must be integrated in pre-service training courses, and the epistemological assumptions underlying different theoretical approaches must be pointed out and discussed.

Issues for debate

1. How different theoretical approaches to the use of historical references into didactics can influence teachers education?

2. Are perspective teachers aware of epistemological assumptions underlying the uses of the history into mathematics education?

3. In pre-service training courses, what is the role of historical and epistemological aspects? Are they integrated with other components?

4. With reference to perspective teachers' conceptions, what is the status of the historical approach?

Keywords **1.** Historical references. **2.** Epistemological assumptions. **3.** Pre-service teachers. **4.** Secondary.

1. History of mathematics and mathematics teaching and learning

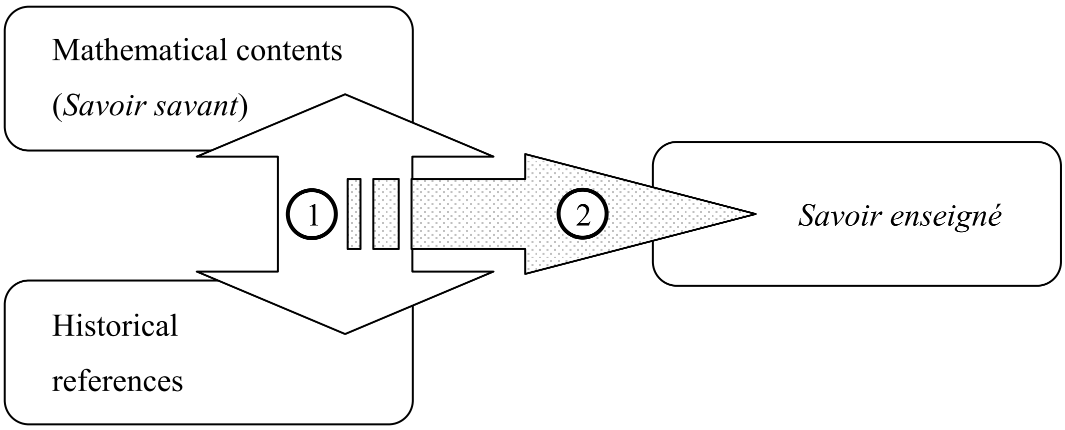
Components of mathematics teachers education are various and relevant either from mathematical points of view or in pedagogical sense. As several studies have pointed out, "the history of mathematics can play a valuable role in mathematics teaching and learning" (Fauvel & van Maanen, 2000, p. XIII): in this paper we are going to discuss the importance of some aspects related to history and epistemology into mathematics education and, in particular, into teachers education.

The role of the history into didactics is a major issue of the international research in mathematics education (Furinghetti & Radford, 2002). According to some theoretical frameworks, the relevance of historical elements is mainly referred to secondary schools curricula, at least if the use of historical references is considered in an explicit sense and with metacognitive purposes; but in our opinion main issues raised by this approach are quite general, so the historical and epistemological education of teachers should be considered with reference to all school levels.

The use of the history into mathematics education links psychological learning processes with historical and epistemological issues (Radford, Boero & Vasco, 2000), and this link can be ensured by epistemology, as explicitly noticed by G. Waldegg (see: Waldegg, 1997).

Concerning the features of the interaction between history of mathematics and educational practice, a wide range of views and experiences can be examined. For instance, different levels can be considered with reference to teaching and learning processes: a first one is related to anecdotes presentation and it can be useful in order to strengthen pupils' conviction (as underlined in: Radford, 1997); higher levels bring out multidisciplinary relations and metacognitive possibilities connected to historical references (see for instance: Furinghetti & Somaglia, 1997).

In order to present different possibilities of using history into mathematics education, let's now consider the following representation (some terms by Chevallard are employed: Chevallard, 1985):



Of course this representation is just a schematic outline: clearly, for instance, the passage from the *savoir savant* to the *savoir enseigné* is not simple. However we can notice that there are two important sets of connections to be analysed and studied:

- connections (1) between mathematical contents and historical references;
- connections (2) between mathematical content-historical references and the knowledge presented by teachers to pupils in classroom practice (after the *transposition didactique*).

In order to clarify the situation and to stimulate discussion, it is important to underline that **aforementioned uses of the history into didactics do not reflect just practical educational issues: as we shall see, they imply some epistemological assumptions** (we shall follow: Radford, 1997; Radford, Boero & Vasco, 2000). For instance, the selection itself of historical data to be presented in classroom practice is epistemologically relevant: of course mathematics curricula must be related to classroom behaviours, but moreover such selection reflects some epistemological choices by the teacher. Several important problems are related to the interpretation of historical data: this is frequently based upon our cultural institutions and beliefs (see: Gadamer, 1975). So connections (1) and (2), previously visualised, must be referred to epistemological assumptions and this aspect must be carefully pointed out in teachers education.

As a matter of fact, perspective teachers can be induced to apply historical knowledge to classroom practice according to an ingenuous approach: for instance the educational introduction of a topic would take place just by the ordered presentation of all the historical references related with it (the "history of the concept"), at the beginning of the classroom presentation. By that we should recognise an introductory relevance to historical references and, sometimes, their possibility to stimulate metacognitive reflections. Such choice implies some epistemological assumptions; and this is not the one and only way of using history into mathematics education.

However often the role of the history into didactics is not considered only from an introductory point of view: a parallelism between the historical development of a concept and the cognitive growth is sometimes explicitly (Piaget & Garcia, 1989) or implicitly considered: as a matter of fact, frequently a new concept is encountered by mathematicians in history in operative stages, for instance in problem solving activities; it will be theoretically framed many years or several centuries later, assuming the features that we (nowadays!) consider typical of mathematical objects (Giusti, 1999). A parallel evolution can be pointed out in the educational field: often the first contact with a new notion takes place in operative stages (Sfard, 1991). It follows that historical knowledge must be considered as a primary component of teachers education: in fact pupils' reactions are sometimes very similar to reactions noticed in mathematicians in history (Tall & Vinner, 1981) and such correspondence can be a very important tool for mathematics teachers.

Of course, this parallelism cannot be considered without a theoretical framework and it would be necessary to theorise further the above given first description (Bagni, forthcoming): as a matter of fact, such parallelism brings us to take into account relevant epistemological problems. A major issue, as we shall see, is related to the interpretation of history: for instance, is it correct to present the history as a path that, by unavoidable mistakes, obstacles overcoming and critical reprises, finally leads to our modern theories? What is the role played by social and cultural factors that influenced historical periods? Mathematical contents deal with non-mathematical context, too. It is necessary to overcome a merely evolutionary perspective: **knowledge cannot be considered absolutely, according to a classical teleological vision; in fact, it must be understood in terms of cultural institutions** (Radford, 1997). Let's briefly present some theoretical frameworks.

According to the "epistemological obstacles" perspective (Brousseau, 1983), one of the most important goals of historical studies is finding problems and systems of constraints (the so-called *situations fondamentales*) that must be analysed in order to understand existing knowledge, whose discovery is connected to the solution of such problems (Radford, Boero & Vasco 2000, p. 163). Obstacles are subdivided into epistemological, ontogenetic, didactic and cultural ones (Brousseau, 1989) and such subdivision points out that the sphere of the knowledge is considered isolate from other spheres. This perspective is characterised by other important assumptions (Radford, 1997): the reappearance in teaching-learning processes, nowadays, of the same obstacles encountered by mathematicians in the history; and the exclusive, isolated approach of the pupil to the knowledge, without social interactions with other pupils and teachers (Brousseau, 1983). With reference to the above-presented schematic picture, we can summarise epistemological assumptions as follows:

(1) knowledge exists and represents the best solution of relevant problems;
epistemological obstacles recur either in history or in educational practice;

(2) the sphere of knowledge is separated from educational and cultural spheres;
pupils approach knowledge individually.

The assumptions needed by the "epistemological obstacles" perspective are very important; the crucial point is the following (Gadamer, 1975; Radford, 1977): **it is impossible, nowadays, to see historical events without the influence of our modern conceptions.** This remark is fundamental, and it must be carefully kept into account in the introduction of the matter in teachers education.

We previously underlined that the "epistemological obstacles" approach is not the one and only perspective according to which we can consider the use of the history in classroom practice. As a matter of fact, we can face up to the mentioned problems so we can explicitly accept the presence of our modern point of view: in other words, we can awarely take into account that, when we look at the past, we connect two cultures that are "different [but] they are not incommensurable" (as noticed in: Radford, Boero & Vasco, 2000, p. 165). Concerning the nature of mathematics, "the historical approach encourages and enables us to regard mathematics not as a static product, with *a priori* existence, but as an intellectual process; not as a complete structure dissociated from the world, but as an on-going activity of individuals" (Grugnetti & Rogers, 2000, p. 45; see also the important "voices and echoes" perspective by P. Boero: Boero & Al. 1997).

According to the socio-cultural perspective by L. Radford, knowledge is linked to activities of individuals and, as we above noticed, this is strictly related to cultural institutions (see for instance: Radford, 1997); knowledge is not built individually, but into a wider social context (Radford, Boero & Vasco, 2000, p. 164). The role played by the history must be interpreted with reference to different socio-cultural situations (Wartofsky, 1979) and moreover it gives us the opportunity for a deep critical study of considered historical periods (Furinghetti & Radford, 2002). With reference to the above-presented picture, we can summarise epistemological assumptions as follows:

(1) knowledge is related to actions required in order to solve problems;
problems are solved within the socio-cultural contexts of the considered periods;

(2) knowledge is socially constructed;
cultural institutions and beliefs of their own culture influence pupils.

2. History of mathematics and mathematics teachers education

Concerning teachers education, the general question is the following: **what is the knowledge needed by a teacher in order to present some mathematical topics according to an historical perspective, and how such knowledge can be proposed to perspective teachers?** Of course this component must be linked with other basic mathematical, educational, didactical and pedagogical aspects of teachers education, and such links are connected to some fundamental epistemological assumptions whose consideration requires perspective teachers' awareness.

Several questions deserve researchers' attention: for instance, what should new entrants to teaching expect to do as mathematics teachers, with reference to the historical approach? In other words, should a perspective teacher expect to teach mathematics by using history? If so, why and how? We shall try to suggest some answers to these questions and to stimulate discussion.

Clearly the assumed level of mathematical competence at entry to pre-service training courses must allow the comprehension of the considered topics, from a mathematical point of view. A good comprehension must be achieved with reference to historical elements, too: so perspective teachers must have an effective knowledge of the history of mathematics. An important factor to be considered is the historical (and epistemological) competence of perspective teachers: **different university courses in different countries grant very different levels of importance to history of mathematics.**

Concerning the aforementioned theoretical frameworks, perspective teachers courses can be influenced either by the epistemological obstacles approach or by the socio-cultural approach. In general:

- if a perspective teachers course deals **exclusively with the history of mathematical contents**, it can make reference to the epistemological obstacles approach (or to similar approaches). Concerning, for instance, the history of a particular mathematical topic, it is possible:
    - to propose to perspective teachers the available references connected to the topic;
    - to look for the epistemological obstacles;

- to analyse the reactions, in the history, to such obstacles in order to present the related knowledge to the pupils.

- if a perspective teachers course deals **either with the history of mathematical contents, either with non-mathematical factors**, it can be referred to the socio-cultural approach. In this case, for instance, we present some historical conceptions of the considered topic in their socio-cultural contexts (as previously noticed, the selection of data is epistemologically relevant: Radford, 1997, p. 28): such exacting choice can cause a remarkable reduction of the treated topics, but it allows the development of an integrate knowledge.

In our opinion, historical knowledge is not enough: perspective teachers must develop such knowledge in the sense of its correct educational use. With reference to Radford's approach previously presented, **history of mathematics cannot be cut off from a wider historical knowledge**, taking into account social, cultural, philosophical elements. So teachers education would consider a comprehensive group of historical aspects: an "internalist" history, so a conception of the development of mathematics as a pure subject, isolated from non-mathematical "external" influences, is hardly useful in mathematics education (Grugnetti & Rogers, 2000, p. 40) and it brings, as noticed, to epistemological problems.

So perspective teachers must be induced to reflect upon the mathematical knowledge: for instance, resolving the old dichotomy of invention versus discovery, we can state that mathematics is either *invented* either *discovered*, as summarised in the following table (which is referred to: Grugnetti & Rogers, 2000, p. 45):

Activities (problem solving)		With reference to
answering problems	*Invention* of new concepts	human creativity and social activity
finding (theoretical) solutions	*discovering* of new theorems and proofs	modes of thought and logical patterns of mathematics

Concerning educational roles played by historical references, the awareness of this situation is a fundamental issue for teachers education: in the third column of the previous table, the role of (mathematical and non-mathematical) context is fundamental. In our opinion, **mathematical knowledge, contents and expressions are essentially influenced by the cultural contexts of the periods in which they were conceived, with reference to different cultural institutions and beliefs, and this is the crucial issue.** The connection between mathematics and socio-cultural context is fundamental: mathematical concepts and techniques are not just "tuned in" to the features of the frame of mind of considered periods: the role of non-mathematical elements is relevant and deep (Radford, 1997 and 2003).

The importance of mentioned elements must be considered not only from the theoretical point of view: they concern classroom practice, too. In particular, we shall briefly present some important examples that can be considered in teachers education:

(a) History of mathematics can be a valuable guideline for the construction of the curriculum. For instance, traditionally a first course in Analysis progresses in the following order:

sets, functions $\rightarrow$ limits $\rightarrow$ derivatives $\rightarrow$ integration

while the historical development occurred in reversed order (Hairer & Wanner, 1996, p. V):

Cantor, Dedekind $\leftarrow$ Cauchy $\leftarrow$ Newton, Leibniz $\leftarrow$ (Archimedes), Kepler, Fermat

Is this a good track to follow? Can the restoration of the historical order (as partly suggested, for instance, in: Apostol, 1977) be educationally effective? And what are the influences of non-mathematical elements? **Teachers training courses can compare different curricula** and point out advantages/difficulties of different approaches.

(b) Mathematics teachers often consider an important issue: the "rigour". But what do we mean by that? Formal correctness must always be investigated in its own conceptual context and not against contemporary standards, in order to avoid the imposition of modern conceptual frameworks to works based upon different ones. For instance, this remark has important consequences dealing with educational uses of original sources: when we consider historical proofs in classroom practice, we often *rewrite* them according to our standards (Barbin, 1994). As noticed, probably this is unavoidable: but it is epistemologically meaningful and all mathematics teachers must always keep it in mind. **Teachers training courses can compare different approaches to the same concept by using original sources, too,** and point out that the different levels of formal correctness reflect our modern point of view.

(c) Uses of representation registers are influenced by different historical periods and paradigms, with different social knowledge structures and different beliefs; in fact there is not a single register of a given kind: the nature of a register depends on the community of practice in question. **Teachers training courses can underline that when a teacher introduces a concept by historical references, he or she must underline that employed expressions are related to the cultural institutions of considered periods**: for instance, the dynamic and the static ideas of limit, as formulated in different historical stages, are encompassed by different semiotic registers (Bagni, forthcoming).

3. Conclusions

The history provides us important educational opportunities: apart from basic levels (for instance, relating historical anecdotes to pupils or introducing a topic by some historical references), we examined the possibility of a metacognitive reflection and the possibility to achieve a wide socio-cultural knowledge of a period. In our opinion, these possibilities are indivisibly linked: in fact the transfer of knowledge from history to didactics cannot be stated just by analogy, but it needs a wider cultural dimension keeping into account mathematical and non-mathematical elements (Radford, 1997) and explaining how philogenetic processes are related to ontogenetic ones. The above-presented approach requires an effective epistemological training of perspective teachers.

With reference to the systems through which mathematics teachers are educated, mathematical knowledge, didactical competence, pedagogical skills and historical knowledge cannot exist in isolation: in our opinion, it is meaningless to state that pre-service training courses should prioritise one of the aforementioned elements as their major concern. **It is necessary to develop an integrated knowledge: the role of the history into didactics must be regarded as one of the components of teachers education; explicit epistemological assumptions grant the connection between mathematical contents, historical elements and their educational uses** (Waldegg, 1997). Perspective teachers must consider and discuss awarely such epistemological assumptions.

Let's finally notice that mathematics teachers are, as previously seen, teachers in a general sense, too; so a correct socio-cultural historical perspective can be connected with the general aims of education service: "the history can assert properly cultural, human, social values of mathematics" (Pizzamiglio, 2002, p. 5, translation is ours).

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