

## EVERT W. BETH

*Almelo, 1908 - Amsterdam, 1964*



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### *Brief scientific biography*

Evert Willem Beth was born in Almelo (eastern Netherlands, near the Dutch-German border) on 7 July 1908 (so in 2008 we can mark his centenary birth anniversary). His father, H.J.E. Beth, had studied mathematics and physics at Amsterdam University and worked as teacher in mathematics and physics in secondary schools (VAN ULSEN, 2000); Evert studied mathematics and physics at Utrecht University, and also studied both philosophy and psychology. His 1935 Ph.D. was in philosophy (Faculty of Arts).

In 1946, Beth became professor of Logic and Foundations of Mathematics in Amsterdam (more precisely, in 1946 he was appointed to a part-time professorship; in 1948 this became a full professorship; it is worth noting that he was the first professor of Logic and Foundations of Mathematics in the Netherlands). Apart from two brief interruptions (in 1952 he worked as a Research Associate at the University of California in Berkeley, with A. Tarski, and in 1957-1958 he taught as a Visiting Professor at Johns Hopkins University in Baltimore), Beth worked in Amsterdam continuously until his death (HEYTING, 1966).



Amsterdam University

Beth was a mathematician, logician and philosopher whose work principally concerned the foundations of mathematics. His name is often remembered with reference to semantic tableaux: the tableau method was devised independently by Beth (1955), Hintikka (1955) and Schütte (1956) and later developed by Smullyan (1968). Essentially, this important method is dual to Gentzen's natural deduction (1934) and it is considered by many to be intuitively simple, particularly for students not acquainted with the study of logic. In fact, Gentzen's method is a systematic search for proofs in tree form, while the tableau method is a systematic search for refutations in upside-down tree form. Beth himself underlined:

"At least three different methods of deduction are known today and are more or less currently applied in research: Hilbert-type deduction, Gentzen's natural deduction, and Gentzen's calculus of sequents. [...] In point of fact the three methods must rather be considered as different presentations of one and the same method" (*Formal Methods*, 1962, XII).

With the semantic tableaux, Beth explored different areas: classical logic, modal logic and intuitionistic logic; in combination with this method, Beth made a proof-theoretic variant: the deductive tableaux (HEYTING, 1966; VAN ULSEN, 2000). More generally, Beth's main contributions to logic were the definition theorem, semantic tableaux and the Beth models. The foundation of his work was Gentzen's extended Hauptsatz, the subformula theorem and Tarskian model theory. During the last period of his life (1960-1964), Beth tried to make his logical research subservient to a wide range of applications, e.g. the study of language, theorem proving, mathematical heuristics and translation methods in natural languages (VAN ULSEN, 2000).

Beth was the main founder of the Netherlands Society for Logic and Philosophy of Science, and he was also active in the organization of the International Association for Logic and Philosophy of Science (HEYTING, 1966). From the educational viewpoint it is worth highlighting the book published in 1955 entitled *L'enseignement des mathématiques*, by Beth, Choquet, Dieudonné, Lichnerowicz, Gattegno and Piaget: these Authors were the founders in 1950 of CIEAEM (Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques) and organized many international meetings (for instance: 1950: Relations entre le programme mathématique des écoles secondaires et le développement des capacités de l'adolescent; 1951: L'enseignement de la géométrie dans les premières classes des écoles secondaires; 1951: Le programme fonctionnel: de l'école maternelle à l'université; 1952: structures mathématiques et structures mentales; 1953: les relations entre l'enseignement des mathématiques et les besoins de la science et de l'industrie; 1953: les rapports entre la pensée des élèves et l'enseignement des mathématiques; 1954: les mathématiques modernes à l'école; 1955: l'élève face aux

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mathématiques. Une pédagogie qui libère).

Beth was a member of the Central Committee of the International Commission on the Teaching of Mathematics (ICMI) from 1952 to 1954. He did the greater part of the editorial work for the series "Studies in Logic and the Philosophy of Mathematics" founded at his initiative. Beth's merits were rewarded by his election in 1953 to the membership of the Royal Dutch Academy of Science and by a honorary doctorate in the University of Gent, conferred on him in 1964, when he was already too ill to travel to Gent in order to receive it (HEYTING, 1966). Evert W. Beth died on 12 April 1964.

Up

### *Quotations relevant to mathematics education*

Beth approach to research in mathematics education was very interesting and profound; let us quote Jean Piaget:



Jean Piaget

"A logician friend of mine, the late Evert W. Beth [...] for a very long time [...] was a strong adversary of psychology in general and the introduction of psychological observations into the field of epistemology, and by that token an adversary of my own work, since my work was based on psychology. Nonetheless, in the interests of an intellectual confrontation, Beth did us the honour of coming to one of our symposia on genetic epistemology and looking more closely at the questions that were concerning us. At the end of the symposium he agreed to co-author with me, in spite of his fear of psychologists, a work that we called *Mathematical and Psychological Epistemology*.

[...]. In his conclusion to this volume, Beth wrote as follows:

*The problem of epistemology is to explain how real human thought is capable of producing scientific knowledge. In order to do that we must establish a certain coordination between logic and psychology.* This declaration does not suggest that psychology ought to interfere directly in logic - that is of course not true - but it does maintain that in epistemology both logic and psychology should be taken into account, since it is important to deal with both the formal aspects and the empirical aspects of human knowledge" (PIAGET, 1970, 1),

It is worth noting that Beth and Piaget gave an important contribution to research in cognitive development; in their book (BETH, PIAGET, 1961) they stated that the problems posed by formalisation can in some way correspond with current mental mechanisms. So the logico-mathematical structures leading to formalisation can be considered as the point of arrival of a long genetic process.

In the "Preface" to the book (1962) *Formal Methods: an Introduction to Symbolic Logic and to the Study of Effective Operations in Arithmetic and Logic* (Table of contents of this work: I. Purely implicational logic. II. Full sentential logic. III. Theory of quantification, equality, and functionality. IV. Completeness of elementary logic. V. The formalization of arithmetic and its limitations. VI. The theory of definition. VII. On machines which prove theorems), written in Amsterdam (October, 1961), Beth stated:

"Many philosophers have considered logical reasoning as an inborn ability of mankind and as a distinctive feature in the human mind; but we all know that the distribution of this capacity, or at any rate its development, is very unequal. Few people are able to set up a cogent argument; others are at least able to follow a logical argument and even to detect logical fallacies. Nevertheless, even among educated persons there are many who do not even attain this relatively modest level of development. According to my personal observations, lack of logical ability may be due to various circumstances. In the first place, I mention lack of general intelligence, insufficient power of concentration, and absence of formal education. Secondly, however, I have noticed that many people are unable, or sometimes rather unwilling, to argue *ex hypothesi*; such persons cannot, or will not, start from premises which they know or believe to be false or even from premises whose truth is not, in their opinion, sufficiently warranted. Or, if they agree to start from such premises, they sooner or later stray away from the argument into attempts first to settle the truth or falsehood of the premises. Presumably this attitude results either from lack of imagination or from undue moral rectitude. On the other hand, proficiency in logical reasoning is not in itself a guarantee for a clear theoretic insight into the principles and foundations of logic. Skill in logical argumentation is the result of congenital ability combined with practice; theoretic insight, however, can only arise from reflection and analysis" (p. X).

A meaningful educational statement is the following:

"Lack of formal education can, of course, be remedied, but hardly by the study of logic alone" (p. X).

Moreover:

"[The student] should become acquainted both with the semantic and with the purely formal approach to the notions, the problems, and the results of logical theory. A dogmatic attitude with respect to the different aspects of logic will easily result if the elements of logic are taught in a narrow spirit. [...] Each one-sided approach leaves part of the material more or less in the dark. [...] It should not be forgotten that later on it is extremely difficult to overcome the bad effects of a narrow-minded initiation" (p. XII).

The importance of historical aspects was frequently underlined by Beth; for instance:

"Recent discussion on the foundations of mathematics and physical science cannot be fully understood without reference to their historical and philosophical background. These discussions for the greater part originate not merely from the results of contemporary scientific research in themselves, but rather from the incompatibility of these results with certain preconceived philosophical doctrines" (in: *Critical Epochs in the Development of the Theory of Science, The British Journal for the Philosophy of Science*, 1, 1, 27).



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