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THE INFLUENCE OF TEXTS' MENTAL IMAGES UPON PROBLEMS' RESOLUTIONS

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ABSTRACT

As regards pupils' approach to problem solving, several Authors stated that pupils dealing with a given problem create a mental model of the proposed situation; but a recent work proved that the full possibility to imagine a situation does not help pupils. In this paper (dealing with pupils aged 13-16 years), we show that, sometimes, this full possibility can constitute a real obstacle to the acceptation of a correct resolution.

PROBLEMS AND MENTAL IMAGES

According to B. D'Amore and B. Martini, "when pupils solve a problem whose text is a written one, they first create a mental model of the situation described in the text itself [...] It is important to analyse if it is necessary to have *detailed* mental model of situations described in texts in order to solve given problems" (D'Amore & Martini, 1997, p. 156; in this paper translations are ours).

So what happens when a pupil deals with a given problem?

Several researches show that it is not easy to answer briefly to such question. As previously underlined, it seems that pupils, dealing with a problem, create a mental model of the situation (see for instance: Zan, 1991-1992). Of course, the correct construction of this mental model is very important in order to achieve a good resolution of the considered problem; some Authors stated that if a problem deals with common and well known situations (for instance, if it is directly referred to common objects, easy to be imagined) then the construction of its model is easy, so its resolution can be rather simple (for instance, let us quote: Vergnaud, 1985; Paivio, 1986; Johnson-Laird, 1988). But in a recent work by D'Amore (1997) this statement was deeply discussed: let us briefly summarise such research.

Three groups of pupils of 5th class of Italian Primary School (aged 10-11 years) were asked to solve the same problem; in the proposed texts, different words were inserted: *Pencils*, *Orettoles* and *Przxetqzyw*; in the case of the word *Pencils*, the described context was easily imaginable. It is not so when the word *Orettoles* is used (what does it mean?); however *Orettoles* is a word which "sounds rather well", in Italian. The last word, *Przetqzyw*, is quite unknown (D'Amore, 1997 and 1999).

As regard correct resolutions, success percentages were approximately the same: so different possibilities to imagine all details of represented situations did *not* influence pupils' performances. How should we interpret such result? In this work, we don't want to propose a real conclusion: in fact the debate is open. We just underline that the possibility to imagine a situation *in all its details* did not seem necessary to help problem solving.

We supposed moreover that, in some cases, the possibility to imagine all details of a situation can constitute an obstacle for some pupils: our work is devoted to the analysis of this possibility.

METHODOLOGY OF OUR WORK

We proposed to some pupils of 3rd class of Italian Middle School (25 pupils aged 13-14 years), of 1st class of Italian High School (*Liceo scientifico*, 26 pupils aged 14-15 years) and 2nd class of High School (*Liceo scientifico*, 23 pupils aged 15-16 years) a classical problem in two different versions: as we shall see, one of them just deals with the geometric situation; in the latter one, there is an "environment" in order to suggest its interpretation.

Mathematical curriculum of all considered pupils was a traditional one; in particular, they knew the Pythagoras' Theorem and its applications to problems of Euclidean Geometry.

We divided every class into two groups, A and B; we gave the following card A to every pupil belonging to groups A:

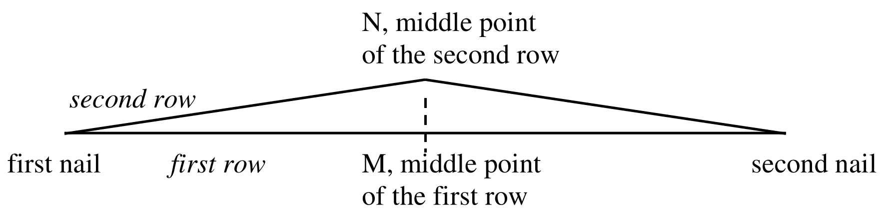
Card A - Problem

The length of the basis AB of an isosceles triangle ABN is 1 000 000 m; the sum of its sides AN, BN is 1 000 001 m; find the length of the height NM.

We gave the following card B to every pupil belonging to groups B:

Card B - Problem

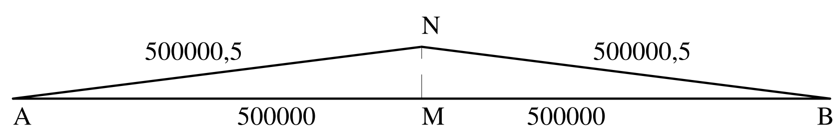
Let us tie a row to two nails very far, say… 1000 km; let us imagine to use a row whose length is exactly 1000 km: so this row will be tight. Then let us tie to the same nails another row, whose length is 1000 km and 1 m; so this second row is a bit longer than the distance between the nails and it will not be tight: in order to stretch it, let us bring the second row in its middle point and let us "raise" such point (see the picture), in order to take it away from the first row, until the second row is completely tight.



Well, how much must we take away the middle point of the second row? Find the distance between the middle point of the first row, M, and the middle point of the second row, N.

Finally, we gave the following card to *all* the pupils:

Resolution



The triangle AMN is a rectangular one (see the angle M). So we can use Pithagoras' Theorem (measures in meters):

$$\overline{MN} = \sqrt{\overline{AN}^2 - \overline{AM}^2}$$
$$\overline{MN} = \sqrt{500000{,}5^2 - 500000^2}$$
$$\overline{MN} = \sqrt{500000}$$
$$\overline{MN} = 707{,}106\ldots$$

So the distance between M and N is 707,106 m (approximately).

In your opinion, is this resolution correct? Why?

Time: 15 minutes. Pupils were allowed to use scientific calculators.

Let us summarise results in the following tables:

Table 1

Card A (total 37 pupils: 13 of 3rd class of Middle School, 13 of 1st class of High School, 11 of 2nd class of High School)

	3rd Middle S.	1st High S.	2nd High S.	Total
correct resolution	10 (77%)	12 (92%)	11 (100%)	33 (89%)
incorrect resolution	2 (15%)	1 (8%)	0 (0%)	3 (8%)
no answers	1 (8%)	0 (0%)	0 (0%)	1 (3%)

Table 2

Card B (total 37 pupils: 12 of 3rd class of Middle School, 13 of 1st class of High School, 12 of 2nd class of High School)

	3rd Middle S.	1st High S.	2nd High S.	Total
correct resolution	4 (33%)	10 (77%)	6 (50%)	20 (54%)
incorrect resolution	5 (42%)	3 (23%)	3 (25%)	11 (30%)
no answers	3 (25%)	0 (0%)	3 (25%)	6 (16%)

So there is a clear, remarkable difference between percentages of pupils that considered given resolution as a correct one: from 89% (card A, table 1) to 54% (card B, table 2). Some pupils justified their answers; let us analyse justifications in the following paragraph.

PUPILS' JUSTIFICATIONS

Apart from miscalculations, the greater part of pupils refusing the resolution, as regards the card B, made reference to answer's unlikeness:

"I think that the proposed resolution is wrong in any calculations: in fact the difference is really very small: a meter and… 1000 kilometres! If I want stretch the row I should need a small distance" (Alberto, 3rd class of Middle School).

In some pupils' opinions there are "hidden mistakes", or "tricks" and "large numbers that cause errors". Pupils that accepted the proposed resolution (once again with reference to the card B), too, showed uncertainty and perplexity:

"It seems to me impossible that a single meter causes a difference of more than 700 meters! I controlled all the calculations and they are correct: so I think that the resolution can be accepted" (Mattia, 1st class of High School).

The following remark, too, is interesting:

"I liked a lot Mathematics when I was a Middle School pupil; but now I can't solve many exercises, and surely I don't like it! There is always an hidden trick, and many times I can't see it" (Giovanna, 1st class of High School).

Let us now quote C. Fiori and C. Pellegrino, that notice: "Mathematics has got the really strange property to inspire extremely contrasting opinions: pupils love it or hate it, they consider it very easy or quite difficult [...] All the pupils, however, have a strong conception of Mathematics" (Fiori & Pellegrino, 1997, p. 428, referred to: Furinghetti, 1993).

DISCUSSION

Let us notice first that our test considered a small number of pupils (from the statistic point of view, moreover, we did not consider a particular sample): so, in order to avoid any over-interpretations, it would be necessary to investigate students' conceptions by further tests, administered to many students.

We pointed out a clear trend: pupils that considered the "abstract" problem (card A) did not underline that proposed data seem rather unrealistic, so they accepted the proposed resolution. They interpreted the figure just in a geometric context.

On the contrary, many pupils that considered the problem with reference to a practical situation (card B) interpreted practically the same data, so they immediately pointed out their unlikeness. F. Furinghetti notice that the visualisation can constitute an effective help to pupils' intuition: "Intuition must be strengthened and visualisation is often a good tool to do it. However, as stated in Shama & Dreyfus, 1991, 'visual' does not mean 'easy'; in Presmeg, 1986 visualisation is associated to gifted pupils" (Furinghetti, 1992, p. 94; see moreover: Kaldrimidou, 1987; Duval, 1994 e 1997; as regards intuition, see: Fischbein, 1983, 1985 e 1987).

From the affective point of view, too, many pupils considered with perplexity the correct resolution and, sometimes, they refused it (as regards affective factors, let us quote for instance: Zan, 1995; Pellerey & Orio, 1996; D'Amore & Giovannoni, 1997). Moreover, it is interesting to consider the nature of pupils' mistakes, for instance with reference to proportion problems, too (Gagatsis & Kafidas, 1995).

We can conclude that D'Amore (1997) clearly proved that the full possibility to imagine a situation does not help pupils; now we state that, sometimes, this full possibility can constitute an obstacle to the resolution (or, as in the examined case, to the acceptation of the correct resolution).

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