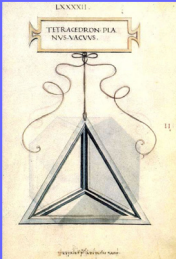


CERME-6, Lyon, January–February 2009

**A few words about Working Group 4
“Algebraic Thinking”**

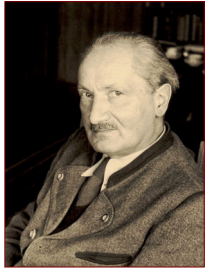


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**“Positive sciences”: a quotation (1927)
by Martin Heidegger (1889–1976)**

- According to Heidegger (*Wegmarken*, 41) we can consider two kinds of science: **the pure science of being** (philosophy) **and some positive sciences**:
- «It is evident from the idea of science as such, insofar as it is understood as a [human] possibility, that there are two basic possibilities of science: sciences of beings, of whatever is, or ontic sciences; and *the science of being, the ontological science, philosophy.* Ontic sciences in each case thematize a given being that in a certain manner is always disclosed *prior* to scientific disclosure. **We call the science of beings as given – of a *positum* – positive sciences».**

**A positive science
and its *positum***



- So the term “positive” refers to a *positum*, a given being that is partially known in the pre-scientific stage, with reference to a particular positive science.
- In Heidegger’s perspective what is scientifically knowable is given in advance by a “truth” which is not graspable by positive sciences: **beings constitute the *positum* of the science, so every particular science (including mathematics) is a “positive” science in itself.**

**A positive science
and its *positum***

- Let us now consider mathematics (and algebra) more closely. We must take into account that, of course, **mathematics is not properly a philosophy**, and, generally, doing mathematics (doing algebra) does not strictly request ontological reflection.
- In order to consider mathematics as a positive science, it may be necessary **to point out its *positum***. So can we describe some given beings, partially known in the pre-scientific stage, about which algebra works? And how? This is important with reference to education: the role of the *positum* (and of its historico-cultural background) can be relevant to educational choices.

**Algebra
and language**

- Connections between the formation of a mathematical (algebraic) object and its expressions in mathematical languages are important issues and deserve a careful discussion (see for instance Sfard’s theoretical model).
- For instance, we can ask ourselves whether thinking and speaking in general, in particular mathematical thinking and speaking, must be considered **objectification means** (according to Heidegger, «thinking and speaking belong together, form an identity. This identity was testified to long ago, insofar as *λόγος* and *λέγειν* simultaneously signified talking and thinking»: *Wegmarken*, p. 59).

**Algebra
and language**

- It is worth noting that this question is based upon the opposition between a technical conception of the language (for instance following Carnap) and an hermeneutic approach (following Heidegger himself).
- Sometimes the language is considered as something that allows objectification (reification) of procedures.
- This approach is too vague and too simple: in fact, often in mathematics education we use the term **object**, but **it is difficult to find a clear and shared notion of mathematical (algebraic) object** (as noticed by D. Slavit, 1997).

Algebra and language

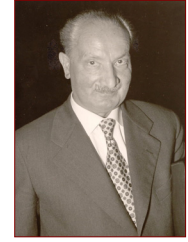
- Heidegger (*Wegmarken*, p. 60) underlined that when we think we do not necessarily represent something as an object. This is true, if ever, in natural sciences.
- So a major point to be discussed is: can we consider mathematics as a natural science?
- **More particularly, do algebra and its didactics need a language which allow the formation of “objects” as previously sketched?**



Algebra and language

- «Is language only an instrument that we employ to manipulate objects? **Is language at all within the human being’s power of disposal?** [...] Is the human being that being that has language in its possession? Or is it language that “has” human beings, insofar as they belong to, pay heed to language, which first open up the world to them and at the same time thereby their dwelling in the world?»

Martin Heidegger (*Wegmarken*, p. 59)



Welcome to WG-4!

Thank you for your kind attention

*Warmest thanks to Janet Ainley
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Jean-Batiste Lagrange*

